$X\setminus\{a\}, X, \emptyset$ offen, $\bigcup_\infty$, $\bigcap_k$ | M offen ⟺ $M = M^\circ$

f stet., $f^{-1}(I)$ offen, wenn I offen

Offenheit v. Ungl. durch f^{-1}

$X\setminus\bar M = (X\setminus M)^\circ$ $\partial M = \bar M\setminus M^\circ$ | M abg ⟺ $M = \bar M$

$X = M^\circ \dot\cup \partial M \dot\cup (X\setminus M)^\circ$

OFFENHEIT U. MENGEN

stetig ⟹ glm. stet.; Min + Max (K auf)

K komp., $A\subset K$ abg. ⟹ A komp

K komp. f stet. ⟹ f(K) komp; ∃ Min, Max

X zshgd, f stet. ⟹ ZWS

X zshgd; $A, B \subset X$ offen, $X = A\dot\cup B$ ⟹ $A = \emptyset \vee B = \emptyset$ | Gebiet: offen + zshgd.

$\mathbb{R}^2 = \gamma(I) \dot\cup G_1 \dot\cup G_2$, G_1, G_2 offen, γ Jordankurve. regulär ⟺ diffbar, $\gamma' \neq 0$

glatt ⟺ γ regulär & C^1

GEBIETE

$\int\gamma = (\int\gamma_1, \dots, \int\gamma_n)$ $\left\|\int_a^b \gamma(t)\,dt\right\| \le \int_a^b \|\gamma(t)\|\,dt$ $L(\gamma) = \int_a^b\|\gamma'(t)\|\,dt$

rektifizierbar ⟺ $L \neq \infty$ γ C^1-Kurve ⟹

$\int_\gamma f\,ds = \int_a^b f(\gamma(t))\underbrace{\|\gamma'(t)\|\,dt}_{ds}$

$\int_\gamma \bar f\,d\bar s = \int_a^b \langle \bar f(\gamma(t)), \gamma'(t)\rangle\,dt$

$d\bar s = \gamma'(t)\,dt$ $\int_\gamma f\,ds = \int_{\gamma^-} f\,ds$ IN-

$\int_\gamma \bar f\,d\bar s = -\int_{\gamma^-}\bar f\,d\bar s$ TEGRATION

γ, β äquiv. ⟺ ∃φ bij., mon. steig. mit $\gamma = \beta\circ\varphi$

γ, β äquiv. ⟹ $\int_\gamma g\,ds = \int_\beta g\,ds$

$\int_\gamma \bar f\,d\bar s = \int_\beta \bar f\,d\bar s$ mit f, g stetig ∫

f bei ā part. diffbar n. x_i ⟺ $f(\bar a + t\bar e_i)$ diffbar bei t=0 DIFFERENTIATION

$\frac{\partial f}{\partial x_i}(\bar a) = \lim_{h\to 0}\frac{f(\bar a + h\bar e_i) - f(\bar a)}{h}$

total diffbar ⟺ ∃ $L_{\bar a}(\bar h)$ | $\text{Mat}[L_{\bar a}] = J_f(\bar a)$

$\lim_{\bar h\to 0}\frac{f(\bar a + \bar h) - f(\bar a) - L_{\bar a}(\bar h)}{\|\bar h\|} = 0$ $\bar f$

f total diffbar bei ā ⟹ f stetig in ā

f total diffbar bei ā ⟹ f bei ā nach allen x_i part. diffbar mit $L_{\bar a}(\bar h) = \sum_{i=1}^n \partial f/\partial x_i(\bar a)\,h_i$

stet. part. diffbar ⟹ total diffbar

$\bar f$ C^1 ⟹ $\bar f$ diffbar | $\bar f$ dbar bei ā, $\bar g$ diffbar bei $\bar f(\bar a)$ ⟹ $\bar g\circ\bar f$ dbar b. ā mit $J_{\bar g\circ\bar f}(\bar a) = J_{\bar g}(\bar f(\bar a))\,J_{\bar f}(\bar a)$

γ dbar in t_0, f dbar in $\gamma(t_0)$ ⟹ $f\circ\gamma$ dbar bei t_0 mit

$(f\circ\gamma)'(t_0) = \langle \text{grad} f(\gamma(t_0)), \gamma'(t_0)\rangle$

f dbar bei ā ⟹ alle Richtungsabl. existieren bei ā mit ... $\partial f/\partial\bar v(\bar a) = \bar v\,\nabla f$ ($\|\bar v\| = 1$)

f dbar ⟹ MWS: $\exists\bar c: f(\bar b) - f(\bar a) = \langle \text{grad} f(\bar c), \bar b - \bar a\rangle$

$F(\bar x, y) = 0$ bei $(\bar a, b)$ lokal n. y auflösbar ⟺ ∃ offene Umgeb. U v. ā, I v. b und $f: U\to I$ mit $N_0(F) \cap U\times I \overset{!}{=} \{(\bar x, f(\bar x)) \mid \bar x \in U\} = \Gamma_f$

F C^1; $N_0(F) \ni (\bar a, b)$, $\partial F/\partial y(\bar a, b) \neq 0$ ⟹ $F(\bar x, y) = 0$ bei $(\bar a, b)$ lokal n. y auflösbar

$\partial f/\partial x_i(\bar x) = -\partial F/\partial x_i(\bar x, f(\bar x)) \,/\, \partial F/\partial y(\bar x, f(\bar x))$ für $\partial F/\partial y \neq 0$

$\bar F$ C^1, $\bar F(\bar a, \bar b) = \bar 0$, $\det\left[\frac{\partial\bar F}{\partial\bar y}(\bar a, \bar b)\right] \neq 0$ ⟹ $\bar F$ bei $(\bar a, \bar b)$ auflösbar AUFLÖSBARKEIT

$\bar f$ C^1, $\det J_{\bar f}(\bar a) \neq 0$ ⟹ $\bar f$ invbar und $J_{\bar g}(\bar y) = J_{\bar f}(\bar g(\bar y))^{-1}$

$\bar g(\bar a) = \bar 0$, $\text{grad} f(\bar a) = \sum_{i=1}^k \lambda_i\,\text{grad}\,g_i(\bar a)$ LAGRANGE FÜR LOKALE EXTREMA

$\bar g(\bar a) = \bar 0$, $\bar 0 = \sum_{i=1}^k \lambda_i\,\text{grad}\,g_i(\bar a)$ | prüfe Ergeb. auf Min/Max f. $\lambda_i \neq 0$

symm. Matrix: $A^T = A$ ABBILDUNG/MATRIX

$q_A(\bar x) = \bar x^T A\bar x$ ≙ zu A geh. quadrat. F.

$\begin{bmatrix} a & b/2 \\ b/2 & c\end{bmatrix} \Rightarrow q(x,y) = ax^2 + cy^2 + bxy$

$\forall \bar x \in \mathbb{R}^n: q_A(\bar x) \ge \lambda_{min}\|\bar x\|^2$

q_A pos. def. $\lambda_{min} > 0$; neg. def: $\lambda_{max} < 0$

indef. $\lambda_{min} < 0 < \lambda_{max}$ | q_A indef: a egal, $\det A < 0$

q_A pd: $a > 0$; $\det A > 0$

q_A nd: $a < 0$; $\det A > 0$

Schwarz: $\frac{\partial^2 f}{\partial x_i x_j} = \frac{\partial^2 f}{\partial x_j x_i}$ wenn f C^2

$H_f(\bar a) = \left[\frac{\partial^2 f}{\partial x_i x_j}(\bar a)\right]$ Hesse-Matrix

f C^2, ā stat. $q = q_{H_f(\bar a)}$ quadrat. Form ⟹ (q pd ⟹ f lok. Min; q nd ⟹ f lok. Max, q indef. ⟹ Sattelp.) q_A pos def. ⟺ alle EW v. A sind pos ⟺ alle Hauptunterdeterminanten sind pos.

q_A neg def ⟺ $-q_A$ pos def.

$f(\bar x) = T_r(\bar x - \bar a) + R_{r+1}(\bar x - \bar a)$; $\bar h = \bar x - \bar a$

mit $T_r(\bar h) = \sum_{k=0}^r \sum_{\bar\alpha\in\mathbb{N}^n, |\bar\alpha| = k} 1/\bar\alpha!\,\frac{\partial^k f}{\partial\bar x^{\bar\alpha}}(\bar a)\,\bar h^{\bar\alpha}$

$\sum_{\bar\alpha\in\mathbb{N}^n, |\bar\alpha| = r+1} \partial^{r+1} f/\partial\bar x^{\bar\alpha}(\bar a + \bar h)\,\bar h^{\bar\alpha}\,\frac{1}{\bar\alpha!}$ TAYLOR

$f(\bar x) = f(\bar a) + \langle \text{grad} f(\bar a), \bar h\rangle + \frac12\bar h H_f(\bar a)\bar h^T$ | $V_3(H) = \frac13\int_\Gamma \bar r\,d\bar F$

Gauß: $\int_B \nabla\bar f = \int_{\partial B}\bar f$ | $\text{grad}\,r^m = m r^{m-2}\bar r$; $\text{div}(r^m\bar r) = (m+n) r^m$

f intbar ⟹ f(?) intbar INTEGRATION

$\left|\int_R f\right| \le \int_R |f|$ $R_1 \cap R_2$ dünn ⟹ $\int_{R_1\cup R_2} f = \int_{R_1} f + \int_{R_2} f$

dünn ⟺ ∀ε>0 ∃ Rechtecke $A_1, \dots, A_n$: $\subset \bigcup_{k=1}^N A_k$ und $\sum_{k=1}^N |A_k| < \varepsilon$ | intbar ⟹ stet. glm.

f stetig ⟹ $F(x) = \int_c^d f(x,y)\,dy$

Fubini: $\int_a^b\int_c^d f(x,y)\,dy\,dx = \int_R f = \int_c^d\int_a^b f(x,y)\,dx\,dy$ für f stet.

g, h stet.; $f = g(x)h(y)$ ⟹ $\int_R f = \int_a^b g(x)\,dx\int_c^d h(y)\,dy$ f, g □

beschr. und $\{f \neq g\}$ dünn ⟹ (f intbar ⟺ g intbar) und $\int_B f = \int_B g$

B proj. in y-Rtg: $B = \{(x,y) \mid a \le x \le b;\ g(x) \le y \le h(x)\}$ gutartig ⟺ B_i projizierbar; $B_i \cap B_j$ dünn f. $i \neq j$

messbar ⟺ $\int_B 1$ exist. | B komp. zshgd, messbar,

proj.bar ⟹ messbar

f stet ⟹ MWS: $\exists\bar b\in B: \int_B f = f(\bar b)\,V_2(B)$

$\int_Q f = \int_{a_{i_n}}^{b_{i_n}}\left(\dots\int_{a_{i_1}}^{b_{i_1}} f(\bar x)\,dx_{i_1}\right)\dots)\,dx_{i_n}$

$\int_{g(a)}^{g(b)} f(x)\,dx = \int_a^b f(g(u))\,g'(u)\,du$

Transf.: $(f\circ g)\,|\det J_{\bar g}|: B\to\mathbb{R}$ intbar

$\int_{\bar g(B)} f = \int_B f(g(\bar u))\,|\det J_{\bar g}(\bar u)|\,d\bar u$

gilt Transf. | beiliegt: $C = \begin{pmatrix} r\cos\varphi \\ r\sin\varphi\end{pmatrix}$

⟹ $V_2(C) = \frac12\int_\alpha^\beta R(\varphi)^2\,d\varphi$ INT

$V_3(C) = \pi\int_a^b \rho(z)^2\,dz$ Rotationskörper

$|\det J_{\bar g}(\bar u)| = r^2\sin\Theta$ (Kugel) ≡ r (Zyl)

$\bar N = \frac{\partial\phi}{\partial u}(\bar b)\times\frac{\partial\phi}{\partial v}(\bar b)$; $\bar n = \frac{\bar N}{\|\bar N\|}$; $V_2(\Gamma) = 2\pi\int_a^b \gamma_1(t)\,\|\gamma'(t)\|\,dt$ (Fläche Rot.)

$\int_\Gamma f\,dF = \iint_B f(\phi(u,v))\,\underbrace{\|\bar N\|\,d(u,v)}_{dF}$

Γ orientierbar ⟺ ∃ $\bar n$ (stet. Vektorfeld) mit $\|\bar n(\bar p)\| = 1$; $\bar n(\bar p) \perp T_{\bar p}$ $d\bar F = \bar n\,dF$

$\int_\Gamma \bar f\,d\bar F = \pm\iint_B \langle \bar f(\phi(u,v)), \bar N_{\phi(u,v)}\rangle\,d(u,v)$

$V_2(\Gamma) = \int_B \|\bar N\|$ f stet., $\exists\frac{\partial f}{\partial x}$ ⟹ Diff, Int vertauschbar $F(x) = \int_{g(x)}^{h(x)} \frac{\partial f}{\partial x}(x,y)\,dy + f(x, h(x))\,h'(x) - f(x, g(x))\,g'(x)$

Green: $\bar f = (P, Q)$ C^1 ⟹ $\iint_B\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)d(x,y) = \int_\gamma \bar f\,d\bar s = \int_\gamma P\,dx + Q\,dy$ $V_2(B) = \int_\gamma -y\,dx = \int_\gamma x\,dy = \frac12\int_\gamma -y\,dx + x\,dy$

Stokes: $\int_\Gamma \text{rot}\,\bar f\,d\bar F = \int_\gamma \bar f\,d\bar s$ ∫

$\int_H(\langle \text{grad} f, \text{grad} g\rangle + f\Delta g) = \int_{\partial H} f\frac{\partial g}{\partial\bar n}\,dF$ | $\int_H(f\Delta g - g\Delta f) = \int_{\partial H}\left(f\frac{\partial g}{\partial\bar n} - g\frac{\partial f}{\partial\bar n}\right)dF$ $A^{-1} = \frac{1}{\det A}\begin{pmatrix} d & -b \\ -c & a\end{pmatrix}$

$\text{div}(\bar f\times\bar g) = \langle \text{rot}\,\bar f, \bar g\rangle - \langle \bar f, \text{rot}\,\bar g\rangle$ | $\text{rot grad}\,f = \bar 0$ | $\text{div rot}\,\bar f = 0$ $A = \begin{pmatrix} a & b \\ c & d\end{pmatrix}$

$\sin^2 = \frac12 - \frac12\cos(2x)$ | $\cos^2 = \frac12\cos(2x) + \frac12$

$\int$: $\sin^2 = \frac12(x - \sin\cos)$ | $1/\sqrt{1-x^2} = \arcsin$

$\cos^2 = \frac12(x + \sin\cos)$ | $-\frac{1}{\sqrt{1-x^2}} = \arccos$

$\cos^3 = \frac{1}{12}(9\sin + \sin(3x))$ | $\frac{1}{1+x^2} = \arctan$

$\sin^3 = \frac{1}{12}(\cos(3x) - 9\cos(x))$ | $\cosh^2 - \sinh^2 = 1$

$\sinh = \frac12(e^x - e^{-x}) = -i\sin(ix)$ | $\cosh x + \sinh x = e^x$